

MATHEMATICS

Information provided in this section includes the following-

- An outline of the Victorian Essential Learning Standards (VELS) in mathematics.
- Stages of Learning-Level 1 to 3.
- Suggested Instructional Strategies and Materials.

See other sources of information on **Mathematics** including-

The Early Learning Checklist in the **Learning Readiness Section**.

Activities in the **Learning Disability Section**.

OUTLINE OF THE MATHEMATIC CURRICULUM VELS STAGES LEVELS 1 TO 3

The structure of this section will be based on the *Victorian Essential Learning Standards (VELS)*, The Domain of Mathematics. While the emphasis will be placed on the early Stages of Learning, (Laying the Foundation) Years Prep. to 4, some of the information should be relevant at the more advanced levels of mathematics learning.

The Dimensions of Mathematics will be explored at Standards-

Level 1-End of Preparatory Year

Level 2-End of Year 2

Level 3-End of Year 4

It is planned to identify potential problem-areas for students generally, but especially for students with a learning disability. To reiterate, learning disabled students have great difficulty processing and retaining directional information, that is, sequential and orientational/positional information. (See **Learning Disability Section** for details).

The Dimensions of the Mathematics Curriculum as identified in VELs are as follows-

- Number
- Space
- Measurement, Chance and Data
- Structures
- Working Mathematically

The abilities to be taught at each of these Dimensions for Levels 1 to 3 will be outlined below. While the role and pervasiveness of **DIRECTIONALITY** (sequence and orientation) as a fundamental feature of early mathematics learning should be clear.

Note: I have highlighted in *italics* the relevant concepts, vocabulary, abilities and processes. As the acquisition and development of the “language of mathematics” forms a seamless continuum of development from **Level 1 to Level 3**, the key concepts and vocabulary introduced in the three levels will be combined.

Levels 1 to 3

Understanding Key Concepts and Vocabulary

The “language of mathematics”

- the language of instruction
- number relationship concepts and vocabulary (e.g., smaller than, bigger than, more, less, equal, every, none, most, least)
- *spatial concepts and vocabulary (e.g., circle, square, triangle, flat, round, big, little, large, round, small, inside, outside)*
- *temporal concepts and vocabulary (e.g., now, after, past, morning, slowly, time, early, quickly, before, clock, o'clock, hands, hours, minutes, seconds, half-past, quarter-past, quarter-to, a.m., p.m., yesterday, tomorrow, day, week, month, season, year, digital, analogue, date, calendar)*
- *directional, positional concepts and vocabulary (e.g., beginning, middle, end, right, left, up, down, in, out above, below, between, behind, in order, forward, backward)*
- quantity concepts and vocabulary (e.g., groups, some, same, empty, full, more, less, least, balance, few, most, as much, how many, how much)
- money concepts and vocabulary (e.g., money, coin, note, cost, buy, cent, dollar, spend, change, worth, much, bought, price,)
- measurement concepts and vocabulary (e.g., long, short, tall, wide, heavy, full, empty, light, area, perimeter, mass, weighs, grams, kilograms, scale, centimetres, millimetres, kilometres, distance, balance, litres, millilitres, number line, ruler, temperature, degrees)
- chance and data concepts and vocabulary (e.g., dice, die, graph, coldest, hottest)
- general mathematics concepts and vocabulary-(e.g., count, number, numeral, figures, words, set, order, group, equal, unequal, same, different, add, plus, take away, share, number sentence, equation, addition, subtraction, multiplication, division, fractions, whole, half, quarter, third, odd, even, prime number)

Note: Obviously, some of these categories of concepts and vocabulary overlap.

LEVEL 1

Learning Focus and Standards

NUMBER

Basic abilities include-

Conservation of number (Piaget, 1965)

Establishing equivalence

Seriation/ordering

Classification

Grouping

Sorting

Forming sets

Specific abilities include-

Rote counting up to 20 and beyond

Counting backwards

Identifying and writing numerals up to 20 (check for reversals)

Associating numeral to number counted

One-to-one correspondence

Reading number words to 20

Understanding ordinal values (first, fifth, tenth, etc)

Recognising symbols-plus, add, minus, equals

Model addition, aggregation-counting and placing items together

Model subtraction, disaggregation- moving apart items

Addition and subtraction by counting forward and backward using numbers from 0 to 20

SHAPE

Identifying basic two dimensional shapes (e.g., circle, square)

Identifying three dimensional solids and objects such as boxes, balls

Recognising interior and exterior of shapes and objects

Sorting geometric objects

Placing and orienting shapes according to simple descriptions such as next to, beside, in front, behind, over, under

Locating items in the classroom and immediate environment

MEASUREMENT, CHANCE AND DATA

Comparing objects

Estimating and simple measurements using informal units (e.g., paper clips)

WORKING MATHEMATICALLY

Activities and play to develop skills in correspondence

Creating and exploring number patterns using counters and other objects

LEVEL 2

Learning Focus and Standards

NUMBER

Understanding place value of the natural numbers 0 to 1000

Counting and ordering numbers to 1000 by 1s, 10s, 100s

Creating number patterns mentally, by hand and calculator

Skip counting by 2s, 5s, 5s from 0 to 100 starting from any natural number (and backwards)

Recognizing patterns when skip counting (forward and backward)

Adding and subtracting one and two-digit numbers by counting on and counting back

Mentally computing simple addition and subtraction calculations involving one-or two-digit natural numbers

Using number facts such as complement to 10, doubles and near doubles

Using commutative and associate properties of addition and multiplication in mental computations (for example, $3 + 4 = 4 + 3$ and $3 + 4 + 5$ can be done as $7 + 5$ or $3 + 9$)

Describing and calculating simple division as sharing such as 8 shared between 4

Describing simple fractions half, third, quarter in terms of equal sized parts of a whole object

Dividing geometric shapes into equivalent parts to develop concept of a simple fraction as part of a whole

Ordering of money amounts in dollars and cents, carrying out simple calculations such as change from small amounts

SPACE

Recognising lines, surfaces and planes, corners and boundaries

Identifying key features of shapes and solids

Identifying familiar two dimensional shapes including rectangles, rhombuses and hexagons

Identifying three dimensional shapes and objects including pyramids, cones and cylinders

Arranging a collection of geometric shapes into subsets
Describing symmetry, asymmetry and congruence of shapes and objects
Investigating symmetry of shapes with mirrors and folding
Drawing simple 2 dimensional shapes and visualising the effect of transformations (e.g., slides, rotations, flips, turns and enlargements)
Specifying location as a relative position including left and right
Interpreting simple network, diagrams and maps
Constructing and following directions, informal maps, diagrams and routes to locations in the local environment

MEASUREMENT, CHANCE AND DATA

Making, describing and comparing measurements of length, area, volume, mass and time using informal units
Recognizing time units-seconds, minute, hour, day, week and month
Investigating basic time patterns and cycles
Learning to tell the time to hours and half-hours using analogue clocks and to hours and minutes using digital clocks
Describing temperature using qualitative terms (for example, cold, warm, hot)
Using terms, litre, metre and kilogram
Recognising key elements of the calendar and placing in sequence days, weeks and months
Investigating natural variability in chance events and order them from least likely to most likely
Conducting experiments
Using pictographs and simple bar graphs to organize and present data
Playing games of chance to recognise and describe variability of outcomes
Using terms such as "unlikely" and "almost certain", "more likely" and "less likely" to describe everyday chance events

WORKING MATHEMATICALLY

Using place value to enter and read displayed numbers on a calculator
Using a four-function calculator including use of the constant addition function and multiply key to check accuracy of mental and written estimations
Undertaking written estimations and approximations and solutions to simple number sentences and equations
Using a combination of everyday language and mathematical statements and symbols to describe their manipulation and play with sets of numbers, shapes, objects and patterns

Modelling and describing daily activities and familiar events using physical materials, diagrams and maps

LEVEL 3

Learning Focus and Standards

NUMBER

Using place value (as the idea that “ten of these is one of those”) to determine the size and order of whole numbers to tens of thousands

Rounding numbers up and down to the nearest unit, ten, hundred or thousand

Using material to develop concepts of decimals to hundredths

Developing fraction notation and comparing simple common fractions such as $\frac{3}{4} > \frac{2}{3}$ using physical models

Developing concepts of equivalent fractions and comparing fraction size

Skip counting forwards and backwards, from various starting points using multiples of 2, 3, 4, 5, 10, 100

Estimating the results of computations

Computing with numbers up to 30 using all four operations

Providing automatic recall of multiplication facts up to $10 \times 10 = 100$

Using written methods for

- *Whole number problems of addition and subtraction involving numbers up to 999*
- *Multiplication by single digits (using recall of multiplication tables) and multiples and powers of ten (for example, 5×100 , 5×70)*
- *Division by single-digit divisor (based on inverse relations in multiplication tables)*

Using algorithms for addition and subtraction of numbers to two decimal places including situations involving money

Applying number skills to everyday contexts such as shopping

Adding and subtracting simple common fractions using physical models

SPACE

Recognising and describing directions of lines as vertical, horizontal or diagonal

Recognising angles are the result of rotation of lines with common end point

Exploring the concept of angles as turn (for example, using a clock face)

Recognising and describing polygons

Recognising and naming three-dimensional shapes such as spheres, prisms and pyramids

Identifying edges, vertices and faces

Visualising and drawing simple solids as they appear in different positions

Using two-dimensional nets, cross-sections and simple projections to represent simple three-dimensional shapes

Producing simple tessellations and puzzles such as tangrams
Locating and identifying places on maps and diagrams
Giving travel directions and describing positions using simple compass directions and grid references on a street directory

MEASUREMENT, CHANCE AND DATA

Estimating and measuring length, area, volume, capacity, mass and time using appropriate instruments
Recognising and using different units of measurement including informal, formal and standard metric measures in appropriate contexts
Reading linear and circular scales in measurement context
Reading digit time displays and analogue clock times at five minute intervals
Interpreting time tables and calendars in relation to familiar events
Planning and conducting chance experiments
Recognising different types of data: non-numerical, separate numbers or points
Using column or bar graphs to display the results of an experiment

STRUCTURE

Using structured material (in tens, hundreds and thousands) to develop ideas about multiplication by replication and division by sharing
Recognising that sharing of a collection into equal-sized parts (division) frequently leaves a remainder
Investigating sequences of decimal numbers generated using multiplication or division by 10
Understanding the meaning of the "=" in mathematical statements and technology displays
Using number properties to support computations (e.g., they use the commutative and associative properties for adding or multiplying three numbers in any order or combination)

Investigating the distributive property to develop methods of multiplication and division by single digit whole numbers
Learning to use and describe simple algorithms for computations
Using simple rules to generate number patterns
Constructing number sentences with missing numbers and solving them
Creating and completing number sentences using whole numbers, decimals and fractions
Listing all possible outcomes of a simple chance event
Using lists, Venn diagrams and grids to show the possible combinations of two attributes

WORKING MATHEMATICALLY

Using mathematical symbols (for example, brackets, division and inequality)
Applying number skills to everyday contexts such as shopping, with appropriate rounding to the nearest cent
Recognising the mathematical structure of problems and using appropriate strategies to solve problems
Developing and testing ideas across the content of mathematical experience. For example in:
Number, *the size and type of numbers resulting from computations*
Space, *the effects of transformations of shapes*
Measurement, Chance and Data, *the outcomes of random experiments and inferences from collected samples*
Testing the truth of mathematical statements and generalizations. For example, in
Number *(which shapes can easily be used to show fractions)*
Computations *(whether products will be odd or even, the patterns of remainders from division)*
Number Patterns *(the patterns of digits of multiples, terminating or repeating decimals resulting from division)*
Shape Properties *(which shapes have symmetry, which solids can be stacked)*
Transformations *(the effects of slides, reflections and turns on a shape)*
Measurement *(the relationship between size and capacity of a container)*
Using a calculator to explore number patterns and check accuracy of estimations
Using a variety of computer software to create diagrams, shapes, tessellations and to organise and present data

SUGGESTED INSTRUCTIONAL STRATEGIES AND MATERIALS

As with the teaching of reading, the teaching of mathematics has not resulted in one particular method or set of material being established as having universal superiority.

In the teaching of mathematics the debate about methods has not been as vociferous or as controversial as has been the case in teaching reading. There seems general

agreement that the methods and/or materials developed by Piaget, Cuisenaire, Gattegno and Dienes continue to have relevance and value.

Learning can be best promoted and reinforced through sound, general teaching attributes including the following characteristics of instruction-

- direct
- explicit
- sustained
- sequential, and
- systematic

The components of a basic Instructional Model, important for learning generally but essential for students with a learning disability, that is, a disability in learning the basic literacy and numeracy skills, are listed below.

Planning Stage

- a task analysis of the process/skill to be taught
- instructional objectives
- necessary concepts
- necessary language
- steps involved, in explicit, sequential order
- identifying student's current knowledge through curriculum-based assessment or formal tests
- rich information concerning student's instructional needs can be found in workbooks: frequently, the precise breakdown in computational skills or understanding is readily apparent, especially if student is requested to "talk through" the strategy used, step by step
- aids to assist understanding and consolidate learning

Implementation Stage

- instructions-teacher "talk"
- instructional procedures-(clear, concise, explicit, direct, sequential)
- aids to establish and consolidate learning
- activities/reinforced practice, including rote drill
- guided practice
- corrective feedback

Revision Stage

- review and revision
- concise recapitulation of instructions
- concise repetition of steps
- set revision and practice exercises

Some General Teaching Procedures

Progression in learning any concept or process should be as follows-

Concrete-basic, practical experiences with objects and other materials

Pictorial- representations in pictorial or diagrammatic form

Symbolic-the symbolic language and the written algorithms of mathematics

Abstract- the higher cognitive skills of problem solving

Establishing and reinforcing the appropriate language of mathematics

Establishing and reinforcing the basic concepts in mathematics

Establishing and reinforcing the correct procedures in mathematics

* the left to right convention in-

- o counting
- o number sentences
- o place value

Establishing and reinforcing, through drill/repetition/practice, basic number bonds (addition and subtraction facts). Establishing and reinforcing basic number facts (multiplication and division)

- drill
- flashcards
- number squares
- tables charts

(Knowing basic number facts and “tables” is best established through a combination of rote learning, identifying patterns and grasping rules)

Establishing and reinforcing the basic algorithms-

CONSISTENCY is critical

- set out the steps in each of the four processes
- have a reference card with the steps identified and arrows to show “direction”
- right to left processing- $+$, $-$, \times
- left to right processing- \div
- top to bottom processing
- determining a set procedure, e.g., the process of subtraction causes significant confusion especially where mixed approaches are used e.g., decomposition/equal addition/borrowing and paying back etc.: again, CONSISTENCY of approach is critical
- watch student perform the process and request student “talks” through the process/steps

Establishing and reinforcing concepts and procedures in common fractions

- use of concrete aids such as real items and Cuisenaire rods

Establishing and reinforcing time measurement-analogue and digital clocks

- basic concepts and language
- concrete activities

Determining the appropriate pace of instruction

Ensuring the student has time to learn

While not covering all areas of the Primary school mathematics curriculum, the above instructional strategies attempt to address some of the major areas of uncertainty, confusion and difficulty for most young students, especially those with a learning disability.

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